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Ad Hoc Briefing Note

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Accounting for Options: Option Pricing Models & Executive Share Options

Contents

OVERVIEW

Introduction

Accounting for Options

Standard setters and many institutional investors (ourselves included) take the view that improvements are needed to existing requirements on the accounting treatment of share options[1], in order to improve the quality of financial reporting. In that context, a draft international standard has been prepared by the International Accounting Standards Board (IASB) based on the following principles:

- when a company makes option awards to executives, it should recognise an expense;
- that expense should be recognised over the relevant period (to vesting); and
- the expense should be measured by reference to fair value.

To estimate the fair value of executive share options, which don't have a quoted market price, companies will be expected to use an option pricing model, "such as the Black-Scholes model or a Binomial model".

Option Pricing Models

Given the above and both the interest and uncertainty that has been expressed about option pricing models, this briefing note reviews the two models that have been referred to and the issue that then arise in relation to executive share options. It aims to provide both an overview and enable a slightly more detailed insight into the arcane world of option pricing and its 'black boxes', if necessary, but hopefully without going so far into it that readers need to reach for their actuary. The Black-Scholes model and the Cox, Ross and Rubinstein ('Binomial') model are the two standard pricing models used in (traded) option pricing. Both are based on the same theoretical foundations, although there are some key differences in how they are applied.

Executive summary

The Black-Scholes Model (see Appendix A)

The Black-Scholes model is used to calculate the theoretical price of a traded European style[2] option using five key inputs[3] as part of a single (albeit complex) linear calculation. It is however, dependent, on a number of assumptions, such as the theory that markets are truly efficient. Its main advantage is speed, which can be crucial in the cut and thrust of the traded options market. The 'price' it produces reflects the amount required by an option writer to both write the option and completely hedge the risk.

The Binomial Model (see Appendix B)

The Binomial model basically solves the same equation, using a computational procedure, that the Black-Scholes model solves using an analytic approach. In doing so it provides opportunities along the way to check for early exercise during the life of the option, by breaking down the time to expiry into component time periods. This enables the model to be used to price American style options[4]. For each period in the process, it is assumed that the share price will move up or down, producing a binomial distribution tree, which represents the possible paths that the share price could take during the life of the option.

The Relationship between the two Models

As noted, the same underlying assumptions underpin both the Black-Scholes and Binomial models, namely that share prices follow a stochastic (random) process. The Binomial model effectively provides discrete approximations of parts of the continuous process underlying the Black-Scholes model. As a result, for European options, the Binomial model will converge on the Black-Scholes model as the number of binomial calculation steps increases. Put another way, the Black-Scholes model for European options is a special case of the Binomial model where the number of binomial steps is infinite.

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[1] : The standard being developed will also apply to other forms of share-based payment.
[2] : European style options can only be exercised on the expiry date
[3] : Underlying share price, strike price, volatility, time to expiration, and the short-term (risk free) rate of return (see Appendix D).
[4] : American style options can be exercised at any time during the life of the option

Probability Analysis (see Appendix C)

Probability analysis is a way of balancing the variables going into the models and factoring in such things as the impact of performance criteria in the valuation of executive share options – a fundamental area of weakness in the core models. It is commonly done using a Monte Carlo simulation exercise or overlay. This uses all of the data used in the core models, plus extra inputs on volatility. While, for example, the linear Black-Scholes model generates just one end value, the simulation performs a specified number of calculations (10,000 or more) which can then be amalgamated to derive a balanced, probability based output. The same principal applies in respect of each of the time periods within a Binomial model.

Changing inputs/variables

With so many potential variable inputs, it is important to have some understanding of how changing those inputs will affect the valuation of an executive share option. The following offer some examples:

- **As distribution widens/(narrows)**
  - an executive share option increases/(decreases) in value
- **As share price goes higher/(lower)**
  - an executive share option increases/(decreases) in value
- **If volatility is higher/(lower)[5]**
  - an executive share option increase/(decrease) in value
- **If interest rates are forecast higher/(lower)**
  - an executive share option increases/(decrease) in value
- **If dividends/coupons/income are higher/(lower)**
  - an executive share option decreases/(increases) in value
- **If a performance target is made more/(less) stretching**
  - an executive share option decreases/(increases) in value
- **If the opportunities to retest performance are more/(less)**
  - an executive share option increases/(decreases) in value

Another property of options is that the longer the term of the option, the more value it has (its 'time value'). This reflects the fact that the longer the time, the wider the range of possible future values for the asset and therefore the higher the probability that you will make money with the option[6]. So an option with a ten year life has greater value than one with a five or seven year life.

Conclusion

Calculating "fair value" for executive share options will never be straightforward, not least because they have features that the standard valuation models do not incorporate. These features relate not only to aspects like duration, but also to conditional elements such as performance hurdles and vesting requirements -precisely the kind of feature that helps make an executive share option an effective performance incentive.

As a result, the use of probability analysis (using a Monte Carlo simulation approach) should be considered to be a particularly attractive, if not essential, method of enhancing the core models (with a strong preference for the Binomial model) to allow for the calculation of a fair value for executive, rather than traded, options.

A critical issue, however, must be the need for common standards and methodologies to ensure accounting for options is not discredited by lack of consistency or suggestions of manipulation and aggressive practice. This is an issue that the International Accounting Standards Board (IASB) does not currently address in its proposals for an International Accounting Standard[7].

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11 March 2003

[5] : See Appendix D
[6] : This holds true until the dividend yield exceeds the risk free rate of return (modelling that points to a cut off after around 16 to 20 years, i.e. its irrelevant for these purposes).
[7] : At the time of writing: Exposure Draft (ED 2 Share-Based Payment) - www.iasb.org.uk

APPENDIX A

The Black-Scholes Model

The Black-Scholes model is used to calculate a theoretical call price (ignoring dividends paid during the life of the option) of traded options[8], using five key determinants of their price: share price, strike price, volatility, time to expiration, and short-term (risk free) rate of return (or interest rate).

The original formula for calculating the theoretical option price (OP) is as follows:

$$OP = SN(d_1) - Xe^{-rt}N(d_2)$$

Where:

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{v^2}{2}\right)t}{v\sqrt{t}}$$

$$d_2 = d_1 - v\sqrt{t}$$

The variables are:

S = share price
X = strike price
t = time remaining until expiration, expressed as a percent of a year
r = current continuously compounded risk-free rate of return
v = annual volatility of share price (e.g. the standard deviation of the short-term returns over one year)
ln = natural logarithm
N(x) = standard normal cumulative distribution function
e = the exponential function

Assumptions of the Black-Scholes Model

1. **Markets are efficient** - This theoretical assumption suggests that people cannot consistently predict the direction of the market or an individual share. Clearly this isn't the case in reality. It implies that the market operates continuously with share prices following a continuously stochastic process where each point is dependent only on the preceding point (Ito's lemma, which is a derivation of the Markov process[9], is used in the modelling).

2. **European exercise terms are used** - European exercise terms mean that the option can only be exercised on the expiry date. American exercise terms allow the option to be exercised at any time during the life of the option, making American options more valuable due to their greater flexibility. This limitation is not a major concern in traded options markets as very few calls are ever exercised before the last few days of their life. The reason is that: (i) when you exercise a call option early you forfeit the remaining time value on the call, to collect the intrinsic value; and (ii) towards the end of the life of a call, the remaining time value is very small, but the intrinsic value is the same.

3. **The risk-free rate of return is used** - The Black-Scholes model, importantly, uses the risk-free rate to represent interest rates remaining constant and known[10]. Although no there no such thing as the risk-free rate in reality, recognised surrogates do exist. In the UK for executive share options we could use the rate of return derived from a five year Government Gilt Strip, in much the same way that the US traded option markets uses the discount rate on U.S. Government Treasury Bills with 30 days left until maturity.

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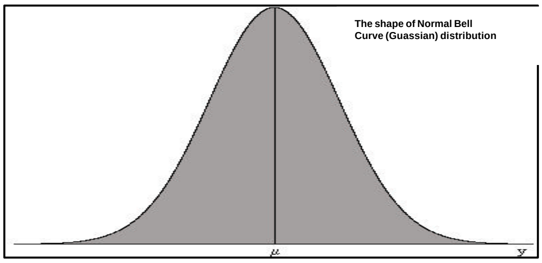
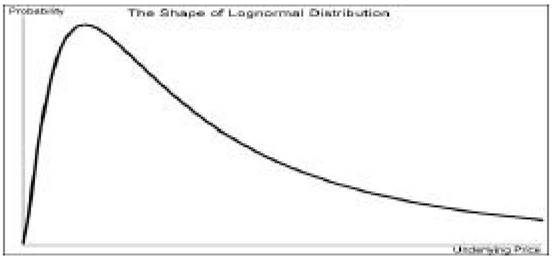
[8] : traded options usually expire after a period of up to 90 days.
[9] : A Markov process is a stochastic process where only the current value of the variable is relevant for predicting the future. The past history of the variable and the way the current value emerge is irrelevant.
[10] : See section on expected return in Appendix D.

4. **No dividends are paid during the option's life** - Most companies pay dividends to their share holders, so this might seem a serious limitation to the model considering the observation that higher dividend yields elicit lower call premiums. In theory you can address this by subtracting the discounted value of a future dividend from the share price. In modelling terms, the common way of managing this is to adjust the risk free rate of return by subtracting the dividend yield.

5. **Returns are lognormally distributed** - The model is based on a normal distribution of underlying asset returns, i.e. the underlying asset prices themselves are lognormally distributed. In practice underlying asset price distributions often depart significantly from the lognormal as, for example, dramatic market moves occur with greater frequency than would be predicted by a normal distribution of returns (both more very high returns and more very low returns).





6. **No commissions are charged** - Usually market participants have to pay a commission to buy or sell options (even floor traders pay some kind of fee, but it is usually very small), but the fees that the 'man-on-the-street' pays is more substantial and could distort the output of the model.

7. **Conditionality** – The model makes no allowance for conditionality, such as requiring the achievement of performance measures.

Related terms you might come across

- **Delta** (δ) : a measure of the sensitivity the calculated option value has to small changes in the share price. A deeply out-of-the-money call will have a delta very close to zero; a deeply in-the-money call will have a delta very close to 1; an at-the-money call is in theory 0.5 (although the higher the volatility the more you move towards 1).[11]
- **Gamma** (γ): a measure of the calculated delta's sensitivity to small changes in share price.
- **Theta** (θ): a measure of the calculated option value's sensitivity to small changes in time till maturity.
- **Vega** (ν): a measure of the calculated option value's sensitivity to small changes in volatility.
- **Rho** (ρ): a measure of the change in option price given a one percentage point change in the risk-free interest rate.

Disadvantages

Clearly a key issue is that much of the theory underpinning the model assumes that there is an efficient market. At a more practical level the Black-Scholes model limitation is highlighted by the fact it cannot be used to accurately price options with an American or executive style exercise, as it only calculates the option price at one point in time (at expiry). It does not consider the steps along the way where there could be the possibility of early exercise of an American option[12] or the different vesting dates associated with conditional elements of an executive share option. Various adjustments are sometimes made to the Black-Scholes price to enable it to approximate American option prices (e.g. the Fischer Black Pseudo-American method) but these only work well within certain limits (they also don't really work well for put options). See also the section on expected return in Appendix D.

Advantages

The main advantage of the Black-Scholes model is speed, it lets you calculate a very large number of option prices in a very short time, which is important in the cut and thrust of the traded options markets.

[11] : With traded options, call option deltas are positive and put option deltas are negative, reflecting the fact that the put option price and the underlying share price are inversely related (the put delta equals the call delta -1).

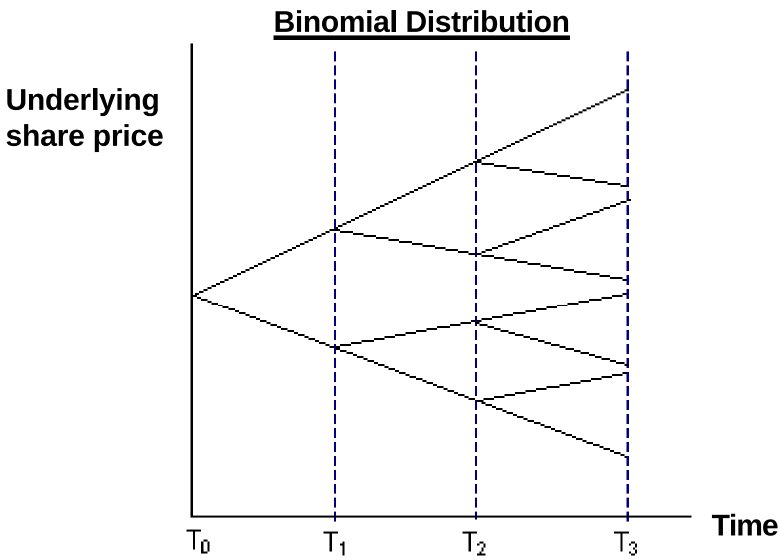
[12] : The exception to this would be an American call on a non-dividend paying asset, where the call is always worth the same as its European equivalent because there is never an advantage in exercising early.

APPENDIX B

The Binomial Model

The Binomial model essentially solves the same equation as the Black-Scholes model, using a computational procedure and, in doing so, provides the opportunity of allowing for early exercise during the life of the option.

The Binomial model breaks down the time to expiration into potentially a very large number of time periods. A series of share prices is produced working forward from the present to expiration. For each period it is assumed that the share price will move up or down by an amount calculated using volatility and the time to expiry. This is a binomial distribution, or recombining tree, of the underlying share prices (see diagram below). The tree represents all the possible paths that the share price could take during the life of the option. At the end of the tree (i.e. at expiration of the option) all the terminal option prices for each of the final possible share prices are known as they simply equal their intrinsic values.



Next the option prices at each step of the tree are calculated working back from the expiry to the present. The option prices at each step are used to derive the option prices at the next step of the tree using risk neutral valuation based on the probabilities of the share prices moving up or down, the risk free rate and the time interval of each step. Any adjustments to share prices (e.g. at an ex-dividend date) or option prices (e.g. as a result of early exercise of American options) are worked into the calculations at the required point in time. At the bottom of the tree you are then left with one option price.

Advantage

The big advantage the Binomial model has over the Black-Scholes model is that it can be used to accurately price American options. This is because with the Binomial model it's possible to check at every point in an option's life (i.e. at every step of the binomial tree) for the possibility of early exercise[13]. With an executive share option different tranches of an option award might vest in tranches in years three, four and then five as tiered performance targets are retested from a fixed base. Where an early exercise point is found it is assumed that the option holder would elect to exercise, and the option price can be adjusted to equal the intrinsic value at that point. This then flows into the calculations higher up the tree and so on.

Disadvantage

The main disadvantage of the Binomial model is its relatively slow speed. Its not a practical solution in the traded options market for quickly calculating a great many option prices.

[13] : This would happen where, due to a dividend or a put being deeply in the money, the option price at that point is less than the its intrinsic value.

APPENDIX C

Probability Analysis (Monte Carlo Simulation)

Probability analysis is a particularly effective way of balancing the variables going into the models and factoring in such things as the impact of performance criteria in the valuation of executive share options – a key weakness of the core models. It is commonly done using a Monte Carlo simulation exercise/overlay.

In practice, an investment return is chosen at random, for each calculation, from the range of probable returns dictated by the expected return and volatility of the asset (you'll note there is still an element of subjectivity). Each of these values is then assigned to distinct valuation bands. The ratio of the number of values in each valuation band, to the total number of trials in a simulation, will determine the probability of occurrence of the value range.

Looking first at the effect on the Black-Scholes model, it does help address that model's linear effect and the potential distortion inherent using a single set of selected inputs. The simulation uses all of the data used in Black-Scholes' linear analysis, plus extra inputs on volatility. While the linear exercise generates just one end value, the simulation performs a specified number of calculations (10,000 or more) which can then amalgamated to derive a balanced, probability based output.

The same principal applies in respect of the Binomial model, with the added advantage that the Binomial model allows you to adjust the assumptions at annual points during the three, four or five year performance period of an executive share option, to reflect changing trends.

APPENDIX D

Volatility & Expected Returns

Volatility

This is one of the key inputs into option pricing, as option prices are very sensitive to changes in volatility, hence the interest in this particular issue in relation to expensing executive share options. Volatility however cannot be directly observed and must be estimated.

Whilst implied volatility (the volatility of the option implied by current market prices) is commonly used, it does not provide a complete picture. In reality an option trader wouldn't rely solely on implied volatility, but would look behind the estimates to see whether or not they are higher or lower than you would expect from historical and current volatility, and hence whether options are more expensive or cheaper than perhaps they should be.

It's a slight over simplification, but basically implied volatility helps give you the *price* of an option in the traded options market, while historical volatility helps give you an indication of its *value.* It can, therefore, be important to understand both. There are a number of ways of calculating historical volatility:

- **Close-Close:** the square root of the mean of the squared deviations of closing prices from a sample. This is the most widely used approach for traded options.

- **High-Low:** uses the formula by Parkinson (1980) which produces an estimate of the volatility using high and low prices from each period in the sample

- **High-Low-Close:** uses the formula by Garman and Klass (1980) which produces an estimate using high, low and closing prices.

- **High-Low-Open-Close:** uses the formula by Rogers and Satchell (1991) which produces an estimate using high, low, open, and close prices.

- **EWMA:** Uses closing prices to calculate volatility using the exponential weighted, moving average model. Includes optional estimating of the smoothing constant (Lambda) using the maximum likelihood method. EWMA is a specific case of the GARCH model, without mean reversion.

- **GARCH:** (Explained further below) This uses closing prices to calculate volatility using GARCH (1,1). The GARCH function also includes a forecasting capability that lets you estimate the volatility for specified periods into the future (eg the volatility one month, two months and three months into the future).

The various calculation types have their advantages and disadvantages. Amongst the un-weighted methods (the first four), the high-low-open-close method is often said to produce the 'best' result. The close-close comes next, but is less statistically efficient, requiring more observations than the others. The high-low and high-low-close approaches are often considered to underestimate the true volatility.

The EWMA method will usually produce a better result than the un-weighted methods. The GARCH model (Generalised Autoregressive Conditional Heteroscedasticity, in case you wondered) is the most sophisticated of the models. It takes into account the phenomenon of volatility clustering, i.e. that during some periods volatility may be higher than normal and during other periods it may be lower.

The GARCH model also incorporates mean reversion: the tendency for volatility after a period of being unusually high or low to move towards a long run average level. The GARCH model calculates the rate at which this is likely to occur from the sample of prices thereby enabling estimates of future volatility by time to be made. Volatility term structures can easily be constructed using GARCH, allowing you to forecast volatility for any number of days into the future. None of the other models can do this as their best estimate of future volatility is simply the most recent estimate of volatility for the asset.

There is one key point to bear in mind about volatility: there is no one right approach for every situation, hence the number of research papers available on the internet, assessing the core models and focusing on issues such as the optimal period over which price volatility should be sampled.

Having said that, while short-term volatility is appropriate for short-term traded options, for executive share options a longer-term volatility rate should be more realistic and representative. In the current volatile markets using the short-term volatility rate should lead to higher expensing costs and push people to wards using a longer-term volatility measure, which would be lower. There is a real need to achieve consistent standards and approaches, otherwise the value of the international standard could end up being partially undermined.

For the layperson, if there is ever a need or desire to experiment with one of the models available on the internet, the most practical option may be to use the Bloomberg or quarterly LBS volatility figures.

Expected Returns (Risk-neutral valuation)

Unlike volatility, which is all important for determining the fair value of an option, views about the future direction of an underlying asset (i.e. whether you think it will go up or down in the future and by how much) are deemed *irrelevant* in the traded options market.

This is one of the key principles underpinning the traded option valuation models. It means that the value of a traded option is its expected future value discounted at the risk-free interest rate. In effect it is assumed that the world is risk-neutral and investors needed no compensation for risk and hence the expected return on all securities is the risk-free interest rate.

As a result the actual expected rate of return of the underlying share (which would incorporate risk preferences of investors as an equity risk premium) is *not* one of the variables in the Black-Scholes model (nor the Binomial model). This means that the theoretical value of a traded option under Black-Scholes is completely independent of the expected growth of the underlying asset (hence it is deemed risk neutral).

More practically, the Black-Scholes price is nothing more than the amount an option writer would require as compensation for writing a call and completely hedging the risk. This just re-emphasises the fact that the nature of the ‘product’ that Black-Scholes is designed to value is materially different to an executive share option. The solution to this is to use Probability analysis (see Appendix C).